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On the Determination of the Axial Position of Mars with respect to the Earth at any Epoch. By Richard A. Proctor, Esq.

Two or three months ago I began the work of re-examining the chart of *Mars* which I formed in 1867, with the view of introducing some of the corrections suggested (1) by the observations of Messrs. Green and Knobel during the opposition of 1873, and (2) still more by the remarkably fine series of drawings made by Mr. Green during the opposition of last autumn. But hearing that many valuable observations had been made by Mr. Trouvelot with the Washington refractor, and by Schiaparelli, at Milan, and having also a wish to go systematically through some of the earlier observations, which are inadequately represented in my present chart, I found it impossible to complete the new chart so soon as I had expected. In fact, I may have to defer the communication of my results to the Society for some few months. In the meantime, however, it occurred to me that it might be well to mention what I proposed to do; in order that any observer, who had drawings of interest and might wish to see the results (if new) incorporated in the proposed chart, might have an opportunity of communicating with me, and perhaps sending tracings of such drawings. It also seemed to me that, as there are many who have drawings which they may wish to compare with the calculated actual aspect of the planet at the epochs when those drawings were made, it might be useful to indicate a comparatively simple construction by which the projection of *Mars* at any epoch may be obtained if the declination and right ascension of the planet be known at the time of observation. The last figure (fig. 4) of the present paper indicates the actual construction for two cases, the light lines representing the part of the construction common to all cases, the broken light lines representing the construction for one case, the dotted lines that for the other case, the heavy lines representing the deduced position of the axis and equator of *Mars* for the first case dealt with. It is not at all necessary in the actual employment of this construction that the explanation illustrated by the first three figures should be read through; fig. 4 and the letterpress relating to it will suffice amply for that purpose. But for the sake of completeness, and for those students of such matters who may desire to recognise the reasons on which the construction is based, and one way at any rate by which the usual formulæ for such cases can be established, I give some preliminary explanations.

In the first place let it be noted that by a simple construction we can obtain the position of the pole and equator of *Mars* with respect to the pole and equator of our heavens. Thus we have, from Oudemans' elements, which may be regarded as relating to the year 1834—

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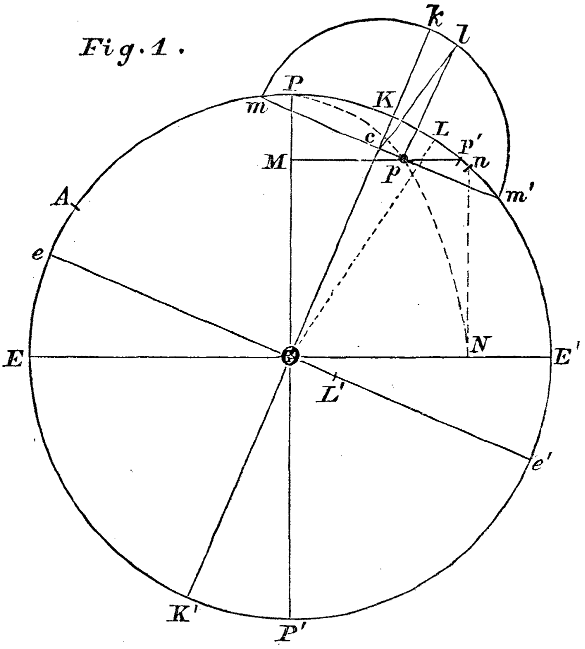
Longitude of pole of *Mars* = 349° 1′ } for Ecliptic, 1834·0.
Latitude = 61 9 }

Now in fig. 1 (as in the others) let POP′ represent the polar axis of the heavens, EOE′ the equator projected into a straight line. In fig. 1 (only) eOe' is the ecliptic similarly projected, the ascending node (♈) being at O on the concavity, the descending node at O on the convexity of the sphere PP′; KOK′ the ecliptical polar axis. Thus POP′ is the projection of the equinoctial colure. Let KAK′L be an arc of 349° 1′, or KL an arc of 10° 59′; then a parallel LL′ to KO would give L′ the point on which a great circle through KK′ and the pole of *Mars* on the heavens would intersect the ecliptic. Also if Km, Km' are arcs of 28° 51′, the straight line mcm' represents the projection of a small circle having K as nearest pole, and passing through the pole of *Mars*. The point p occupied by this pole might obviously be obtained by dividing cm' in p, so that

Fig. 1.



$$cp : cm' :: \mathrm{OL}' : \mathrm{O}e';$$

for then p would lie on an ellipse having KOK′ as major axis, and OL′ as semi-minor axis—this ellipse being the projection of the great circle through KL′K′. But the best way to obtain the point p is to draw the semicircle mkm' on mm', cl parallel to OL, and lp perpendicular to mm'. Or practically the best way is with a protractor to make the angle Kcl (or kcl) equal to 10° 59′, and drop perpendicular lp to mm'.

Having thus obtained p, the northern pole of *Mars* on the heavens, we have only to draw Mpp' perpendicular to PP′ to obtain the arc Pp', which is the north polar distance (I) of the pole of

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Mars. The right ascension of p is obtained by dividing OE′ in N in the same proportion that Mp' is divided in p, and drawing Nn perpendicular to OE′; for then an ellipse having PP′ as major axis and passing through p will pass through N and be the projection of an hour-circle through p, and the inclination of this circle to the equinoctial colure POP′ is measured by the arc Pn. The right ascension of p is, however, equal to the arc PEP′n, and as the ascending node of the equator of *Mars* on the Earth's equator is 90° in advance of the pole of *Mars*, in right ascension, it follows that the arc E′n measures the right ascension, N, of the ascending node of *Mars* on the Earth's equator. This construction carefully made will give results very close to those obtained by the ordinary formulæ of spherical trigonometry, viz.:—

$$\left.\begin{array}{l} \text{R.A. of ascending node of ♂'s equator on ⊕'s equator (N)} = nE' = 47^\circ\ 33'{\cdot}9 \\ \text{Inclination of ♂'s equator to ⊕'s equator (I)} = Pp' = 39^\circ\ 55'{\cdot}6 \end{array}\right\}\text{ in } 1834.$$

To obtain N and I for any other date, $1834 + t$, we make the same correction precisely as for a star in same R.A. and Decl., that is, we correct for the effects of precession, getting for $1834{\cdot}0 + t$

$$N = 47^\circ\ 33'{\cdot}9 + 0'{\cdot}50\,t,$$
$$I = 39^\circ\ 55'{\cdot}6 - 0'{\cdot}23\,t,$$

with amply sufficient approximation.

In passing I may note that the spherical formulæ for obtaining N and I from λ, the longitude of ascending node of equator of *Mars* on ecliptic, and i, the inclination of equator of *Mars* to ecliptic, can be readily deduced from the constructions used in fig. 1. For regarding Mp and OM as projections of Oc and cp (and remembering that angle POK $= \omega$, the obliquity of the ecliptic) we have (putting PO equal to unity),

$$\cos Pp' = OM = \sin i \cos \omega - \cos i \sin \omega \cos \lambda,$$
$$Mp' \cos N = Mp = \cos i \sin \omega + \sin i \cos \omega \cos \lambda,$$

that is,

$$\cos I = \sin i \cos \omega - \cos i \sin \omega \cos \lambda,$$

and

$$\sin I \cos N = \cos i \sin \omega + \sin i \cos \omega \sin \lambda,$$

the required equations.

Let us next consider the construction for obtaining the projections of *Mars* from his known declination and right ascension at any epoch, N and I being known as obtained above, or otherwise.

In fig. 2, POP′ being the polar axis of the Earth, EOE′ the projection of the celestial equator, let pOp' be the polar axis of

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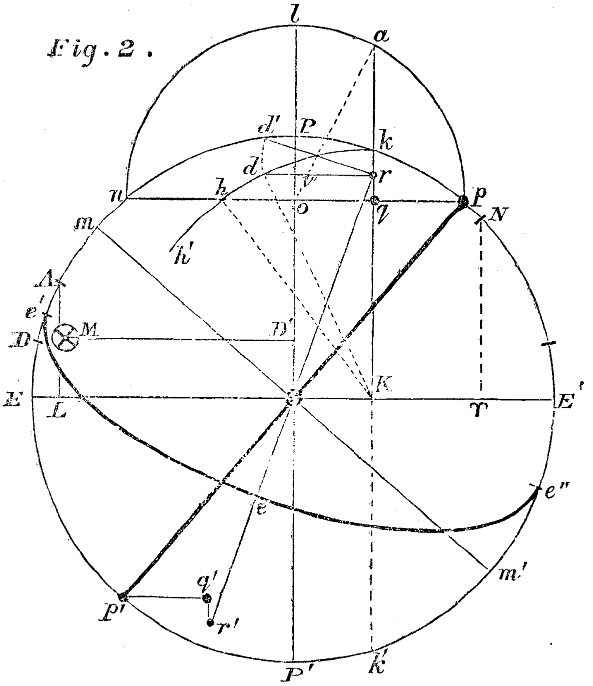
Mars, *m*O*m'* the projection of the equator of *Mars*, the ascending node on the Earth's equator being on the concavity of the sphere PP', at O. Then, if we take arc PN = N, and draw N♈ perpendicular to OE', ♈ on the concavity of sphere PP' is the first point of *Aries*. Now, let

R.A. of ♂ at epoch $= \alpha$,
N. Decl. ,, ,, $= \delta$.

Then, if arc *p*PA $= \alpha$, and arc ED $= \delta$, and AL, DD' be drawn perpendicular to EO and PO respectively, *Mars* lies at M, the intersection of DOD' with an ellipse having PO, OL as semi-axes. The polar axis and equator of *Mars's* globe would of course be situated, in the small disk M, precisely as *p*O*p'* and *m*O*m'*, the polar axis and equator of the Martian heavens, are situated in the disk PP'. And clearly, if we wish to ascertain how *Mars* at M would appear as seen from the Earth at O, we have only to suppose the sphere PEP' so turned that M shall be brought to O; then the sphere PEP' will be presented in precisely the same aspect as the globe of *Mars*, for the Martian celestial axis and equator, *pp'* and *mm'*, are parallel to the axis and equator plane of the globe *Mars* throughout the supposed turning motion. Now, so far as the shape assumed by the equator *mm'* is concerned, the turning might be effected in any way we pleased so long as the centre of M was finally brought to coincidence with O. But as we wish to determine also the position of the polar axis of *Mars* with respect to the meridian, it is convenient to turn the sphere PEP' by two motions which leave the axis

Fig. 2.



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PP′ (or rather its projection) unchanged in position. Thus we first turn the sphere about the axis PP′, bringing M to D′, and carrying the Martian pole p to q (on the convexity of the sphere PP′ in this case, since MD′ lies on the concavity of that sphere); and then, secondly, we rotate the sphere about EOE′ till the centre of the globe M is brought from D′ to O, and the Martian pole from q to r. Then is $r\mathrm{O}r'$ the polar axis of *Mars*, $\mathrm{O}r'$ the remoter half, and the equator of *Mars* has the form shown by the half ellipse $e'ee''$, $\mathrm{O}e$, the minor axis, being (by a well-known property of orthogonal projection) equal to rd' drawn perpendicular to $r\mathrm{O}r'$.

Now, let us consider what construction is indicated for determining the true positions of q and r.

The first rotation of the sphere, round PP′, is through an angle measured by the arc $\mathrm{PA} = (\mathrm{N} - \alpha)$, the right ascension of *Mars* from the ascending node of his equator on the Earth's. Thus, if we draw pon perpendicular to POP′, representing the projection of a declination parallel through p, and on the semicircle pln take arc pa measuring the angle $poa = (\mathrm{N} - \alpha)$, aq perpendicular to np gives us the point q. Produce aq to k', giving kk' the projection of a small circle parallel to great circle POP′. Then we have to rotate the sphere PP′ through an angle measured by arc $\mathrm{DE} = \delta$, the declination of *Mars* (north, positive). Hence, if khh' be part of semicircle on kk' cutting np in h, and the arc hd measures the angle $h\mathrm{K}d = \delta$ (on side of h towards k, because obviously the rotation of D′ on concavity of sphere to O will carry q on convexity towards k), then dr perpendicular to $k\mathrm{K}$ gives the true position of the northern Martian pole at r; the southern will be at r', $\mathrm{O}r'$ on $r\mathrm{O}$ produced being equal to $\mathrm{O}r$; q' shows the position of the southern pole at the end of the first rotation. Of course, both q' and r' are on the concavity of the sphere PP′ in the special case dealt with.

It may be noticed that

$$rd = rd';$$

for the square on rd' = rectangle under kr, rk' = square on rd. So that the construction gives $\mathrm{O}e$ $(= rd')$ without occasion for drawing rd'.

Next let us consider the formulæ resulting from this construction. It will be convenient to regard the angle $k\mathrm{K}h$ as an auxiliary angle. Note also that

$$(hq)^2 = kq.qk' = nq.qp = (aq)^2; \text{ or, } hk = aq.$$

What we require is first the angle $\mathrm{PO}r$; and next the angle between the plane $e'ee''$ and the line of sight to O, or the elevation of the Earth above the plane of the equator of *Mars*. The former angle is commonly regarded as positive when r is on the east of PP′, so that in the position of r in fig. 2 this angle is negative. The Earth's elevation above the plane of the equator

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of *Mars* is considered positive when the Earth is north of this plane, as in the case illustrated by fig. 2. Put then

$$\angle \mathrm{PO}r = -p$$

and

$$\text{inclination of } e'e\,e'' \text{ to line of sight to O} = l.$$

Put also

$$\text{auxiliary angle } k\mathrm{K}h = \mathrm{Q}.$$

Then

$$\tan \mathrm{Q} = \frac{hq}{\mathrm{K}q} = \frac{aq}{\mathrm{K}q} = \frac{oa \sin aop}{\mathrm{O}o} = \frac{\sin \mathrm{I} \sin (\alpha - \mathrm{N})}{\cos \mathrm{I}},$$

$$\tan p = -\frac{\mathrm{OK}}{r\mathrm{K}} = -\frac{oq}{\mathrm{K}d \cos d\mathrm{K}k} = -\frac{aq \cot (\alpha - \mathrm{N})}{hq \operatorname{cosec} \mathrm{Q} \cos (\mathrm{Q} - \delta)} = -\frac{\cot (\alpha - \mathrm{N}) \sin \mathrm{Q}}{\cos (\mathrm{Q} - \delta)},$$

and

$$\tan l = \frac{d'r}{r\mathrm{O}} = \frac{dr}{r\mathrm{O}} = \frac{r\mathrm{K} \tan (\mathrm{Q} - \delta)}{r\mathrm{O}} = \cos p \tan (\mathrm{Q} - \delta).$$

These are the required formulæ for finding p and l.

We can also deduce from the construction the following formulæ :—

$$\cos l \sin p = -\sin \mathrm{I} \cos (\alpha - \mathrm{N}),$$

$$\cos l \cos p = \sin \mathrm{I} \sin (\alpha - \mathrm{N}) \sin \delta + \cos \mathrm{I} \cos \delta,$$

$$\sin l = \sin \mathrm{I} \sin (\alpha - \mathrm{N}) \cos \delta - \cos \mathrm{I} \sin \delta.$$

We note that

$$\sin \mathrm{I} \cos (\alpha - \mathrm{N}) = oa \cos aop = oq = or,$$

and

$$\sin \mathrm{I} \sin (\alpha - \mathrm{N}) = oa \cos aop = aq = hq;$$

also radius OP being throughout regarded as unity,

$$\cos l = \mathrm{O}r, \qquad \sin l = d'r = dr,$$

$$\cos l \sin p = -\mathrm{OK} = -or = -\sin \mathrm{I} \cos (\alpha - \mathrm{N}),$$

$\cos l \cos p = r\mathrm{K} =$ projection of hq and $q\mathrm{K}$ on a line through K inclined at an angle $= d\mathrm{K}h$ to $k\mathrm{K}$, towards $\mathrm{K}h$,

$$= hq \sin \delta + \mathrm{O}o \cos \delta$$

$$= \sin \mathrm{I} \sin (\alpha - \mathrm{N}) \sin \delta + \cos \mathrm{I} \cos \delta,$$

and

$\sin l = rd =$ projection of hq and $q\mathrm{K}$ on a line through K perpendicular to the one last mentioned,

$$= hq \cos \delta - \mathrm{O}o \sin \delta,$$

$$= \sin \mathrm{I} \sin (\alpha - \mathrm{N}) \cos \delta - \cos \mathrm{I} \sin \delta.$$

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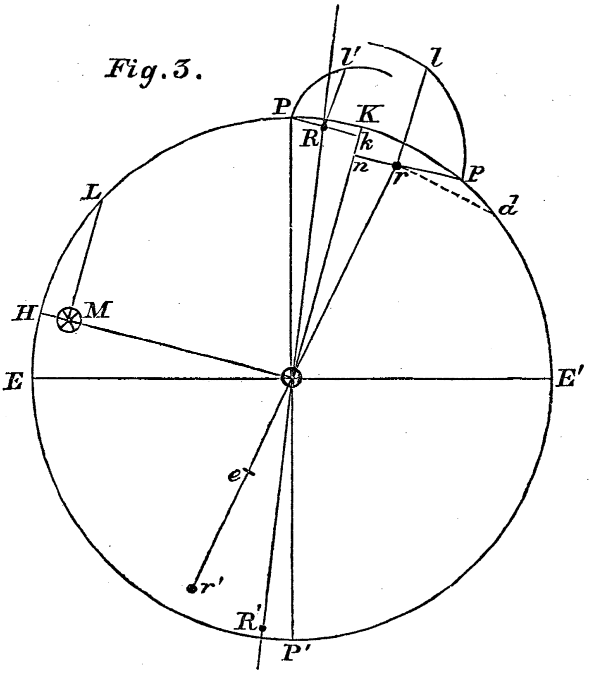
In some cases, where, for instance, the student may have had occasion to determine accurately the projection of the centre of M on the disk PEP′, it may be convenient to carry M to O by a single rotation. Fig. 3 illustrates this method. We join MO; draw ML and OK perpendicular to MO, and pn, Pk to OK. Then taking arcs Pl, Pl' about centres n and k respectively, each measuring the same angle as arc KL, we draw lr and l'R parallel to OK, giving rOr' the position of north Martian pole, and ROR′ the changed position of the polar axis POP′ corresponding to the rotation round KO. The angle ROr is the same as POr from the former construction and the minor semi-axis Oe, the equator of *Mars*, is given by this construction as by the former, since (drawing rd perpendicular to Or)

Fig. 3.



$(rl)^2$ = rect. under segments on either side of r, of chord of which pn is part $= (rd)^2$;

that is

$$rl = rd = Oe.$$

The Actual Construction for Axis and Equator of Mars.

In fig. 4 the actual construction for two cases (with the full result for one only) is shown; and this part of the present Paper is all that the non-mathematical student of the planet need follow.

First, the circle EPE′P′ is drawn with diameters POP′, EOE′ at right angles to each other. Arcs Pp, Pn are taken on either

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side of P, each measuring an angle of $39\frac{2}{3}°$ (I); *pon* is drawn and the half-circle *nlp*; and P′P is produced to *l*.

So much is common to all cases. (This part of the construction may with advantage be carefully drawn in ink on card, to be traced off when wanted; though this will depend on the use to which the constructions are to be applied.) We may add a mark at N, taking PN $= 47\frac{1}{2}° =$ N; but this mark need not actually be used, seeing that it is as convenient to take $(\alpha - \text{N})$ at once.

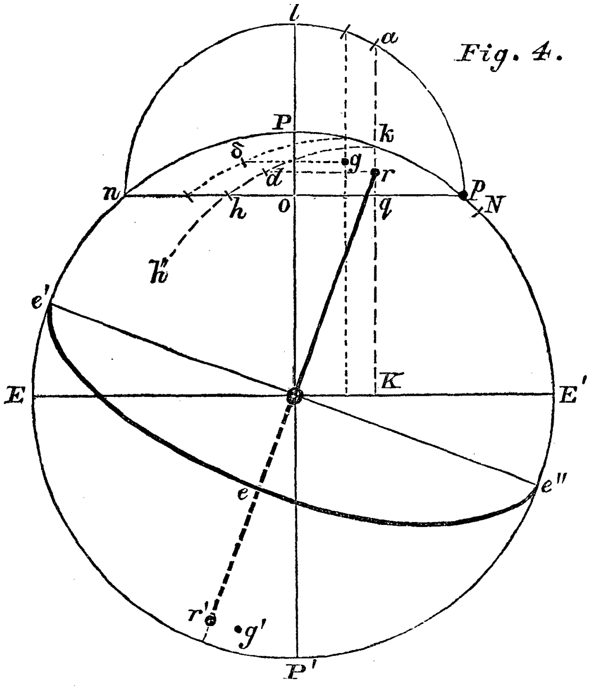

Fig. 4.

Now let the R.A. and Decl. of *Mars* at the epoch dealt with be α and δ respectively.

With a protractor take *pa* measuring angle $(\alpha - \text{N})$ about *o*; and draw *aq*K perpendicular to *pn* and E′E. About K sweep out the arc *khh′*. With a protractor take *hd* measuring arc δ about K. Then *dr* perpendicular to *k*K gives *r* the north pole of *Mars*; *r*O*r′* (O*r′* = O*r*) is the polar axis; *e*O*e′* is the major semi-axis of Martian equator; and O*e* (= *rd*) is the minor semi-axis.

N.B. If $(\alpha - \text{N})$ is between 0° and 180°, *q* lies on the convexity of the sphere PP′. But if $(\alpha - \text{N})$ is between 180° and 360°, we measure this arc from *p* to *n* and then from *n* back to *p* (to avoid drawing the lower semicircle) and *q* lies on the concavity of the sphere PP′. In the former case, northern declinations are measured from *h* towards *k*, and southern declinations from *h* towards *h′*. In the latter case northern declinations are measured from *h* towards *h′*, and southern from *h* towards *k*.

The construction for a second case, $(\alpha - \text{N})$ a few degrees greater and δ also, is shown by the dotted lines. The resulting positions of the poles are shown at *g* and *g′*; and $g\delta$ is the resulting length of the minor semi-axis of the Martian equator. It is not

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necessary, in preparing for an approaching opposition of *Mars*, to make more than five, or at the most seven constructions, the middle one for the day of opposition, since the intermediate positions of the pole and equator of *Mars* can be readily estimated with amply sufficient exactness.

In a future paper I propose to make a few remarks (i) on the completion of the projection of a planet's disk after the position of the poles and equator have been determined; (ii) on the determination of the central areographic longitude for any given epoch; and (iii) on the determination of the centre of illumination.

On the Proper Motion in Right Ascension of η^2 Draconis.

By A. W. Downing, Esq.

My attention was attracted to this star by reading in the preface to the *Nautical Almanac* that the proper motion adopted was that given in Mädler's *Bradley*, and on looking further into the matter I find that from 1848 to 1873 the proper motion in R.A. used in the *Nautical Almanac* was $+0^s.023$, this being the value given in the British Association Catalogue, and since that time $+0^s.005$, taken from Mädler's *Bradley*, has been used. The British Association Catalogue value, $+0^s.023$, has also been adopted in the Greenwich Catalogues since 1845, the Right Ascension of η^2 *Draconis* not being given in the *Fundamenta*, though the Declination is, and consequently the proper motion in the former element has not been computed by Mr. Main or Mr. Stone, whose determinations are based on the *Fundamenta*. The proper motion used in the American *Nautical Almanac* is $+0^s.0024$. As I found that these three authorities differed, I determined to calculate the proper motion in R.A. afresh from the materials afforded by the best available Catalogues.

For this purpose I have, by the kind permission of the Astronomer Royal, used as an assumed R.A. for 1872·0 the place given in the Greenwich Nine-Year Catalogue, so that—supposing the proper motion $=0$— we have for any time

$$\alpha = 16^h\ 22^m\ 15^s.668 + 0^s.8017\ (t-1872) + 0^s.0000939\ (t-1872)^2, \qquad (1)$$

and from this formula I have computed the R.A. for the epoch of each of the Catalogues used.

The following is a list of the Catalogues:—